THE ANALYSIS AND SYNTHESIS OF A FOURTH-ORDER ACTIVE FILTER

by

Kenneth Kiyoshi Yasutome

United States Naval Postgraduate School

THESIS

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Kenneth Kiyoshi Yasutome
Lieutenant, United States Naval Reserve
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ABSTRACT

The realization of linear-phase-shift and constant-time-delay transfer characteristics using computer-aided analysis and synthesis of a fourth-order highpass active filter was the main objective of this investigation.

Two approaches were to be taken in the analysis and synthesis phase Only the one using topologically oriented programs was completed. The second involved the derivation and analysis utilizing the system transfer function.

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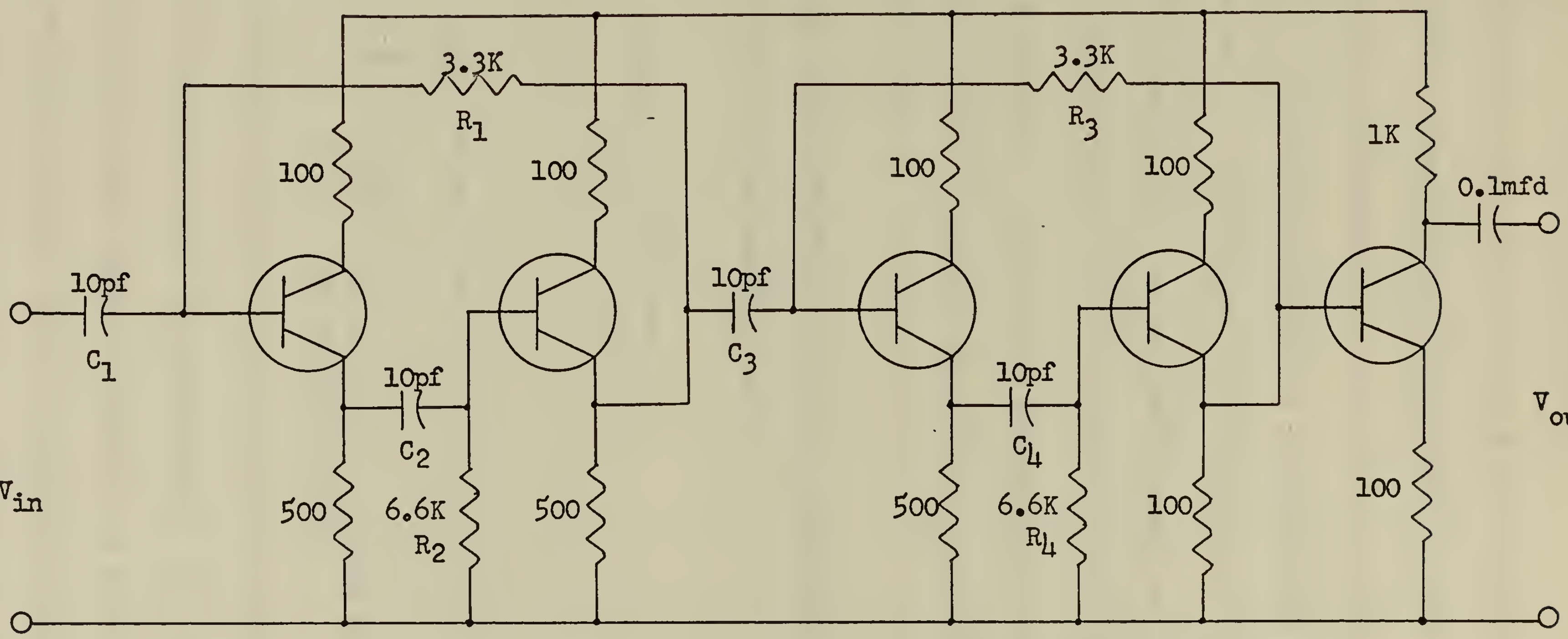
I. INTRODUCTION

The primary objective of this analysis was to determine the parameter modifications to a Sallen-Key type RC active filter to give linear-phase-shift and constant-time-delay transfer characteristics. In 1955 Sallen and Key [Ref. 1] published a catalog of RC active filters. The circuit, Fig. 1, chosen for investigation is a solid-state version of one such filter. The chosen circuit was designed by O. M. Baycura, Associate Professor of Electrical Engineering at the Naval Postgraduate School. The network is actually a fourth-order highpass active filter made up by cascading two second-order filter sections.

Two approaches to the analysis were to be taken, both of which solicit the aid of a digital computer. One method involved network analysis using a topology-oriented program such as ECAP. The second involved the derivation and investigation of the system transfer function. Only the first approach to the problem was completed, due to difficulties encountered with the computer program, and will be the primary topic of discussion.

Upon completion of the computer-aided analysis phase, the filter was constructed and tested using the determined parameters. The actual filter characteristics and those calculated by the computer program were then compared.



All Transistors CDC CS23434

Fourth-Order Highpass Active Filter

Figure 1.

II. CIRCUIT DESIGN

As previously discussed, the circuit is a fourth-order highpass RC active filter made up of two cascaded second-order sections. The second-order filter network is considered by some as the standard building block in filter design. The obvious advantage of a second-order system over higher-order systems is in the simplicity of analysis and optimization. Higher-order filter networks can be realized by cascading individually tuned sections. Figure 2 shows the basic building block of the active filter that was investigated. This circuit configuration is described by Mullaney [Ref. 2] as an RC peaking network and is one of the most widely used filter types.

Assuming the active devices are unity-gain amplifiers, the network is described in Ref. 2 by the transfer function:

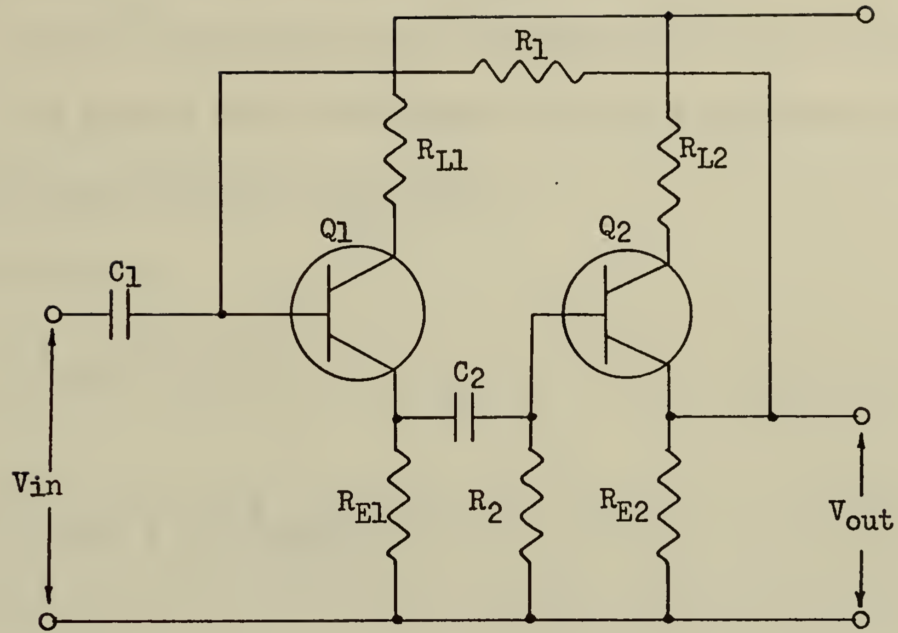
$$G(s) = \frac{K(s/\omega_o)^2}{(s/\omega_o)^2 + s/Q\,\omega_o + 1} \tag{1}$$

where,

ω_o = response break-frequency in radians

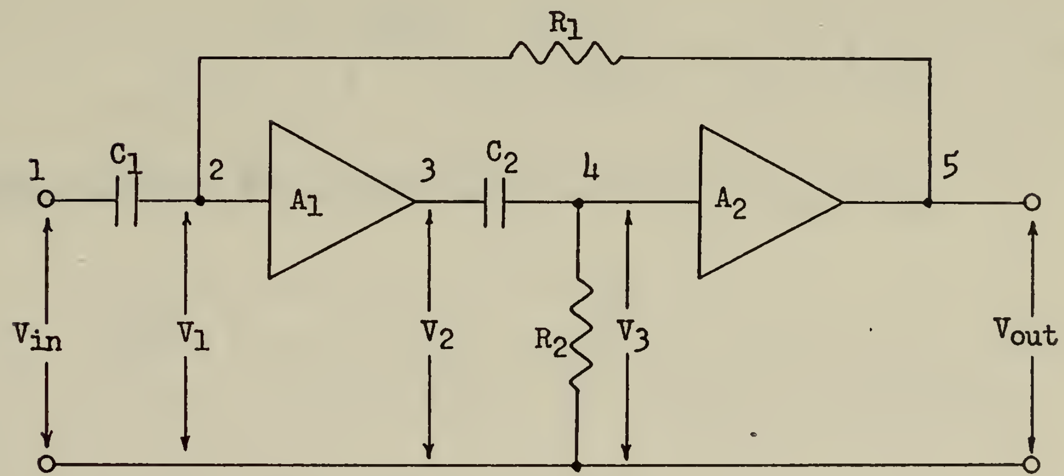
$Q = \dfrac{R_2 C_2}{R_1 C_1}$ = quality factor = amount of peaking of the bandpass filter.

The case with unity-gain amplifiers is a special one and the general case is one in which the amplifier gain is represented by "A". The transfer function describing this case is derived using Fig. 2 with the gains of amplifiers given by A_1 and A_2. These amplifiers are assumed to be ideal and represent dependent sources whose input and output



Second-Order Highpass Filter

Figure 2.



General Second-Order Highpass Filter

Figure 3.

characteristics do not affect the circuit. This leaves the network with the passive complement of two resistors and two capacitors which will result in a second-order polynomial in the denominator of the transfer function. The general form of the transfer function was derived in the following manner using the circuit in Fig. 3:

Starting at Node 5,

$$V_{out} = A_2 \times V_3 \tag{2}$$

where,

$$V_3 = \left(\frac{s}{s + 1/R_2C_2} \right) \times V_2 \tag{3}$$

$$V_2 = A_1 \times V_1 \tag{4}$$

$$V_1 = (V_{in} - V_{out}) \times \left(\frac{s}{s + 1/R_1C_1} \right) + V_{out}. \tag{5}$$

Substituting equations (3) and (4) in (2) and solving for V_1 in terms of V_{out}

$$V_1 = \left(\frac{s + 1/R_2C_2}{s} \right) \times \left(\frac{V_{out}}{A_1 A_2} \right) ; \text{ let } A_o = A_1 A_2 . \tag{6}$$

Substituting (6) in (5) and solving for the ratio of V_{out} to V_{in},

$$\frac{V_{out}}{V_{in}} = \left[\frac{A_o s^2}{s^2 + s[1/R_1C_1(1 - A_o) + 1/R_2C_2] + 1/R_1C_1R_2C_2} \right] . \tag{7}$$

Let

$\omega_1 = 1/R_1C_1$ and $\omega_2 = 1/R_2C_2$

$\omega_o = \sqrt{\omega_1 \omega_2}$.

Then equation (7) becomes

$$G(s) = \left[\frac{A_o s^2}{s^2 + s[\omega_1(1 - A_o) + \omega_2] + \omega_o^2} \right] \tag{9}$$

where the quality factor is

$$Q = \left[\frac{\sqrt{R_1C_1R_2C_2}}{R_2C_2(1 - A_o) + R_1C_1} \right] . \tag{10}$$

The transfer function derived reduces to the form in equation (1) when the amplifier gains, A_1 and A_2, are set to unity. For convenience, the denominator of the transfer function was normalized in the following manner.

$$D(s) = (s/\omega_o)^2 + (s/\omega_o)d + 1$$

where

$$d = 1/Q = \text{the damping factor.}$$

Analysis of second-order systems shows that this configuration is characterized by its two conjugate poles and a pair of zeros at the origin. The roots of the denominator can be found by using the quadratic formula. The roots are given by

$$s = -d/2\omega_o \pm \frac{\sqrt{(d/\omega_o)^2 - 4/\omega_o^2}}{2}$$

which shows that they lie on a circle of radius ω_o and have real parts equal to $-d/2\omega_o$. The system is unstable, (the poles lie in the right-hand side of the s-plane) when the discriminant is positive real and

greater in magnitude than $d/2\omega_o$. The roots will be complex if the parameter d is less than 2 but greater than zero and negative real if d is less than 2. The case of interest in this analysis is the former, where parameter d is greater than zero and less than 2. This filter is categorized as a "low Q" type, [Ref. 2]. The constant ω_o determines the position of the poles in the frequency domain and the parameter d affects the shape of the frequency characteristic. More detailed discussion of this system type can be found in Refs. 1, 2, 3 and textbooks dealing with linear systems analysis.

The component values specified in the circuit of Fig. 1 gives a calculated magnitude for the parameter ω_o of 34.1 Mhz. The variation of the parameter d as a function of the amplifier gain product, A_1A_2, is shown by the graph in Fig. 4 for the same component values.

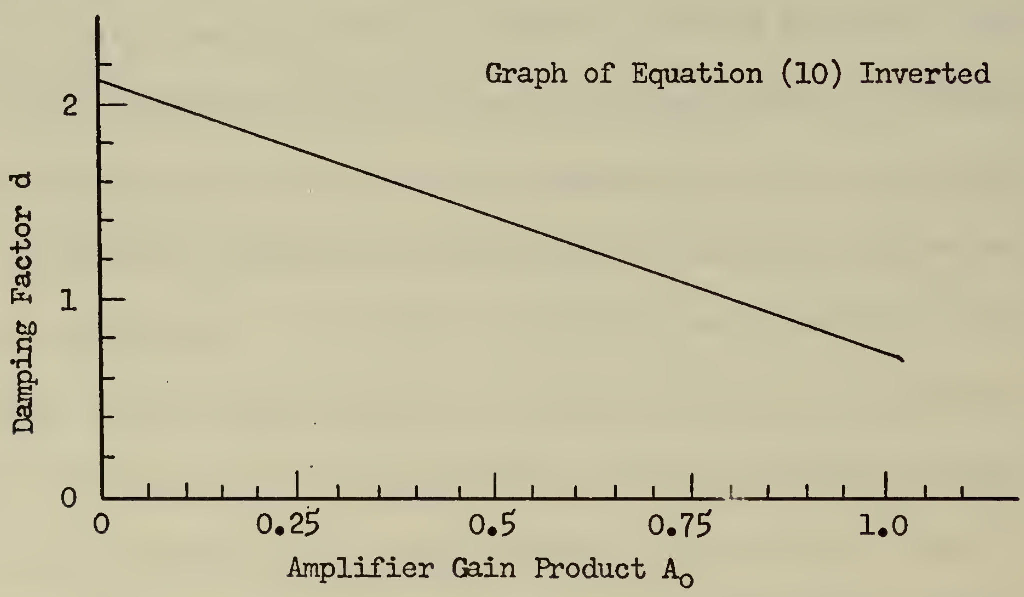


Figure 4. Damping Factor as Function of Gain Product A_o

The nominal value for the measured voltage gain of the emitter-follower amplifier used was 0.6 at 5 Mhz, and assuming the gains A_1 and A_2 equal, a value for the damping factor of 1.7 is obtained from the graph. Using the hybrid-pi model of the transistor, and the analysis procedures discussed in the text by Millman and Halkias [Ref. 47], an attempt was made to calculate the theoretical voltage gain. Difficulties were encountered at this point in determining the necessary parameters. First, published data for the transistor type used was unavailable and, secondly, attempts to measure the high-frequency hybrid parameters failed to produce consistent and useful results. The decision was made to continue the investigation by specifying the parameters for a suitable active device. The hybrid-pi model as discussed in Ref. 4 is shown in Fig. 5 in its high-frequency configuration.

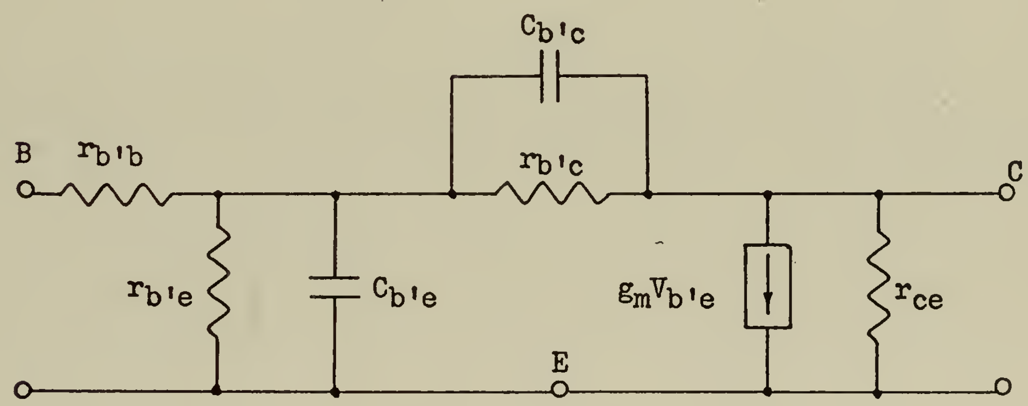


Figure 5. Hybrid-pi Equivalent for Transistor in CE Configuration

After some research on the subject, a model of a transistor studied by Leon [Ref 5.] was chosen as a guideline for specifying the assumed parameters. These parameters are listed in Table 1. With the equivalent

circuit for the active device selected, the equivalent circuit in Fig. 6 for the fourth-order active filter was completed. This is the circuit as initially programmed in the topological approach of the analysis.

Parameter	Magnitude
Base-spreading Resistance r	60 ohms
Input Resistance r	1.4 kilohms
Output Resistance r	25 kilohms
Reverse Feedback Resistance r	10 megohms
Emitter-base Junction Capacitance C	38 picofarads
Collector Junction Capacitance C	3 picofarads
Transistor Transconductance g	90 millimhos

Table 1. Hybrid-pi Parameter Values

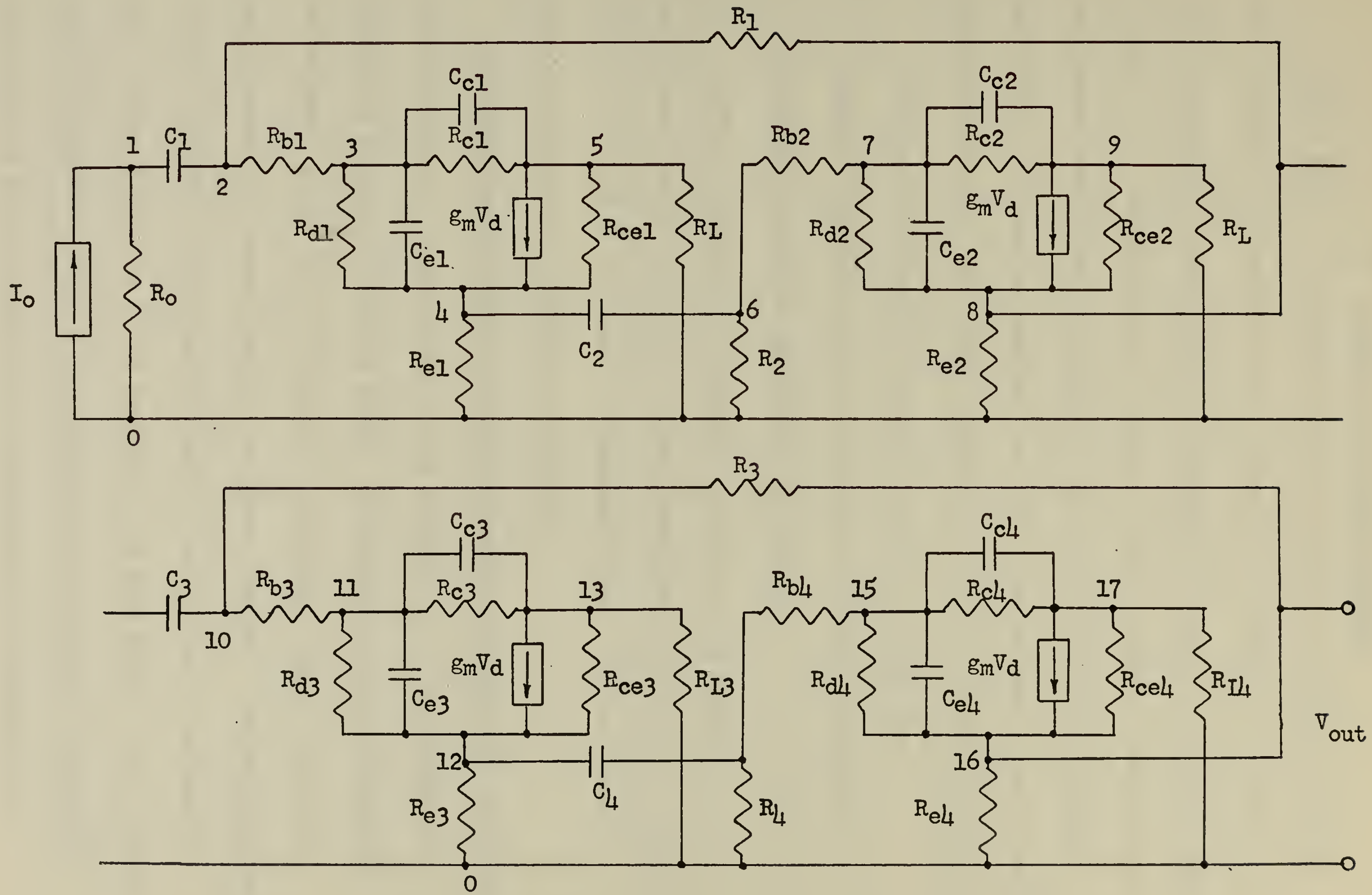


Figure 6. Fourth-order Highpass Active Filter Equivalent Circuit

III. COMPUTER-AIDED ANALYSIS

At this stage, an existing computer program or programs to aid in the analysis of the network was selected. The topological approach to the investigation required that the program be capable of accepting topological circuit description as an input. Since a lumped model with the parameters independent of frequency was used as the active device's equivalent, the program need only be able to solve linear or small-signal transistor circuits. The output capabilities necessary were the following:

Calculate the frequency response and make a Bode plot.

Determine the time delay as a function of frequency and plot.

Solve for the transfer function of the circuit.

Calculate and plot the transient response of the system.

Being restricted to the program library maintained by the W. C. Church Computer Center at the Naval Postgraduate School, the selection of a suitable program was narrowed considerably.

An integrated package of Fortran IV programs called LISA, for LInear Systems Analysis, was located. LISA fulfilled all of the requirements except for the plot of the time delay. The inputs to LISA as described in the user's guide [Ref. 6] may be a topological circuit description, matrix equation, transfer function, or block diagram. The program uses Laplace transform techniques for analysis and the user is allowed to work in the s-plane, or in the frequency and time domain. The topology and the component values of the circuit can be altered by simple programming and they are outputed for comparison and examination.

To use LISA in the analysis of the active filter, the equivalent circuit description is entered in a preformated form as values of R, L, C, mutual inductance, initial conditions, independent current sources, and voltage- or current-dependent current sources. The dependent sources are utilized in modeling active devices in the hybrid-pi or Tee equivalent transistor circuits. The program then forms the nodal matrix equation in the complex frequency variable, $s = \sigma + j\omega$. Also, the functions required by the problem must be defined. These can be voltages, voltage ratios, currents, current ratios, admittances, impedances, determinants, co-factors, or any three-terminal, grounded, two-port network parameters. The program can compute and output as requested poles and zeros, a root locus, frequency and transient response, or sensitivity. The program is capable of computing the Bode response and sensitivity of circuits of up to a maximum of 50 nodes. LISA is also programmed to output error messages for errors detected in the input and compute phases.

After the nodal matrix has been formed, the program is ready to enter the COMPUTE phase. There are three methods by which LISA solves the matrix equations.

The first option, named POLY, solves for the polynomials that define the poles and zeros of the requested functions. POLY can handle systems or order 12 or less with proper input scaling. The limitation of this option is the round-off errors and overflow during floating-point calculations that is caused by its inability to cope with magnitude differences greater than 10^6.

The second method is TRFN which finds the roots of the function directly using the determinant that defines them. The limitations found in POLY are avoided by TRFN since programmed scaling is done at critical locations in the algorithm. The maximum size system sucessfully solved by TRFN is 33.

The third option is ACCA which works in the real frequency domain only. ACCA determines the system response for $s = j\omega$ by evaluating the polynomials in the matrix equation and obtains a system of linear equations with complex coefficient matrix. The system of equations is solved using a modified method of Gaussian elimination. The size of the system that can be handled by ACCA is limited only by the storage available. Systems of the order 50 have been solved by ACCA.

Because of these three options it is possible to check the accuracy of one against another by solving the same problem with each. The outputs of each can be in the form of a listing or can be plotted. The results of the POLY and TRFN options are polynomials and/or roots. To get a frequency-response listing or plot, the option BODE must be used. The ACCA method of solution results in the computation of magnitude and phase over a given range of frequency and the output can be a listing and/or plot.

In addition to the magnitude and phase being computed, the system time delay is also computed. The time delay is found by differentiating the phase function with respect to frequency; that is,

$$\text{Time delay } t(\omega) = \frac{d}{d\omega} \phi(\omega).$$

The output of the time delay is not plotted, but is listed by all three when the Bode plot is requested.

The equivalent circuit in Fig. 6 was programmed on LISA without any scaling. The network consists of one ground node and 16 circuit nodes. The hybrid-pi parameters were renamed for convenience and an independent current source and source resistance were added. The TRFN option for solving the system was chosen and the BODE option, used to obtain the frequency response in the range from 10 Hz to 10 GHz, was requested as a listing and a Bode diagram. The impulse response of the system was requested for the first 150 nanoseconds. The listing and the plots generated could not be included as received due to their large size. The listings will be omitted and the plots, which are made on the line printer, redrawn for proper size for inclusion in this report.

The method of attack followed in the analysis was that of the cascading of individually tuned sections to achieve the desired results. The first second-order filter section was left untouched while the second section was tuned. This meant changing the values of R_1, C_1, R_2, and C_2 which correspond to R_3, C_3, R_4, and C_4 respectively, Fig. 6.

The results of the computation with network topology as specified originally were disappointing. Tuning the second section did not produce the results desired, namely linear phase shift and constant time delay. The fourth-order system was not actually of order four due to the junction capacitances assumed in the active device model. In the analysis done by Sallen and Key discussed in Ref. 1, the active devices are

assumed to be ideal. The amplifiers are assumed to have infinite input impedance, zero output impedance and stable gain. These conditions are also assumed by Mullaney [Ref. 2]. Optimization with the circuit configured as in Fig. 6 seemed futile if not unrealistic at this point. Thus, a decision was made to simplify the problem by eliminating the junction capacitances, programming the network as revised in Fig. 7.

The voltage gain of the amplifier can be calculated using the procedures and relations developed in Ref. 4. Converting the hybrid-pi parameters in Table 1 to hybrid parameters, the voltage gain computes to 0.945 giving a damping factor from graph of Fig. 4 of 0.84. The list of the input statements is given in Table 2 and the frequency and time responses in Fig. 8. In Fig. 8, the values of the resistors R_1 and R_2 are 3.3 kilohms and 6.6 kilohms respectively and the capacitors C_1 and C_2 are equal at 10 picofarads.

The first phase of the search for the optimum values of capacitors, C_3 and C_4, was carried out using the ACCA method since it possessed its own scaling feature. Note that the magnitudes of the components programmed go from a high of 6.6 kilohms to a low of 10^{-11} farads. Even with its own scaling program, ACCA encountered difficulties in computation. Negative time delays were found computed on several runs. There was no correlation of this problem with the order-of-magnitude difference between C_1 and C_3, and C_2 and C_4. Using ACCA, negative time delays were computed when the values of all the capacitors were set equal to 10 pf and then one pair, C_1 and C_2, was held at 10 pf and the

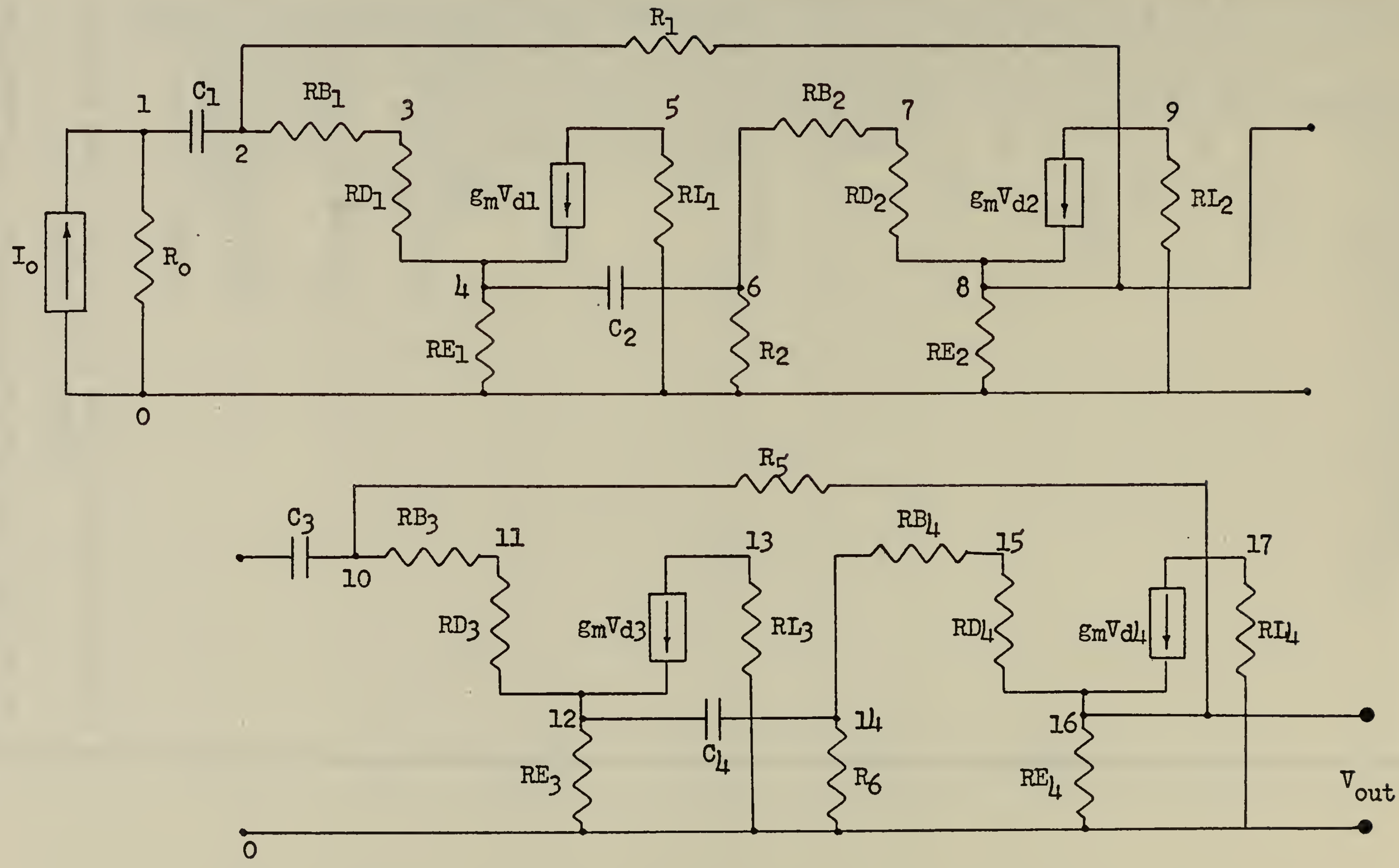


Figure 7. Fourth-Order Highpass Active Filter - Low Frequency Equivalent

Table 2 LISA Input Statements

```
TITLE, FORTH-ORDER HIGHPASS ACTIVE FILTER
NOTE, HYBRID-PI TRANSISTOR EQUIVALENT CIRCUIT
READ,CIRCUIT
IO,0,1,.01
RO,0,1,.1
C1,1,2,1E-11,TOL=.05
RB1,2.3.60
RD1,3,4,1.4E3
R3,4,3,500
TGM1,5,3,4,.09E-3
RL1,5,0,100
C2,4,6,1E-11,TOL=.05
R2,6,0,6.6E3
RB2,6,7,60
RD2,7,8,1.4E3
R4,8,0,500
TGM2,9,7,8,.09E-3
RL2,9,0,100
R1,8,2,3.3E3
C3,8,10,1E-11
RB3,10,11,60
RD3,11,12,1.4E3
R7,12,0,500
TGM3,13,11,12,.09E-3
RL3,13,0,100
C4,12,14,1E-11
R6,14,0,6.6E3
RB4,14,15,60
RD4,15,16,1.4E3
R8,16,0,500
TGM4,17,15,16,.09E-3
RL4,17,0,100
R5,16,0,3.3E3
DEFINE,VOUT=V(16)
COMPUTE,TRFN,VOUT
DATA,FREQ=20,10,10.0,Hz
COMPUTE,BODE,VOUT
PPLOT
LABEL,C1=C2=C3=C4=10 pf
READ,DRIVER
IMPULSE,0,10.
DATA,TIME=0,1.5E-7,1E-9
COMPUTE,TRANSIENT,VOUT
PPLOT,TIME,VOUT
LABEL,VOUT FOR IMPULSE
EXIT
```

Frequency Range: 10 Hz to 10 GHz in 10 decades with 20 points per decade.

Time Interval: 0 to 150 nanoseconds at 1 point per ns.

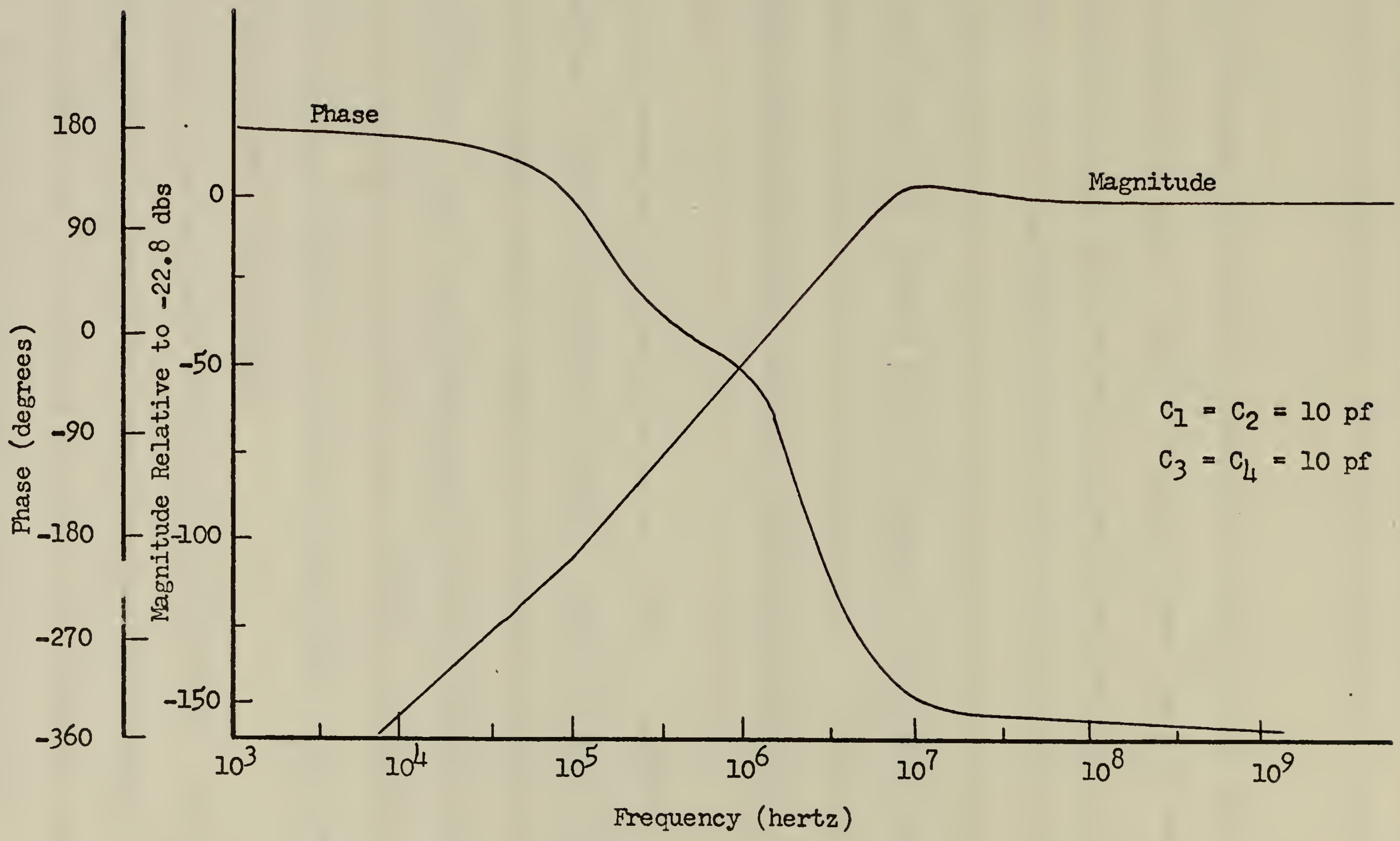


Figure 8 Frequency Response - Original Circuit

other pair changed to 2 pf. When this phenonemon occurred, the negative times were confined to the region well below the passband. Also, associated with the negative times were output magnitudes of less than 10^{-6}.

The TRFN method of solution in combination with the BODE option was next checked for this error. Negative time delays were found to occur when using this method also, but not in all the cases that were encountered using ACCA. Still no correlation could be found between capacitor magnitudes and the phenomenon. Since TRFN has no programmed scaling feature, several runs were repeated with the component magnitudes scaled. Improvement was noticed, but it was not overwhelming. The negative time delays were found to occur with about the same pattern seen in the ACCA runs.

The conclusion was that there must be some order-of-magnitude limit that the program can handle successfully. Just as with the option POLY, the subroutines could be bothered by the same limitations that plague the former. Many root-finding programs have difficulties when the order of magnitudes of the roots becomes very large or the time constants in the system differ by many orders of magnitudes. This is what most likely occurred in this analysis.

Eliminating those parameter values that are troublesome, the range of optimumization was narrowed to between one and two picofarads. The value at the lower end, 1 pf, results in computation of negative time delays. No difficulties were encountered using any higher values. The

results of the last phase topological analysis approach are shown in Figs. 9 and 10 with the values determined listed in Table 3.

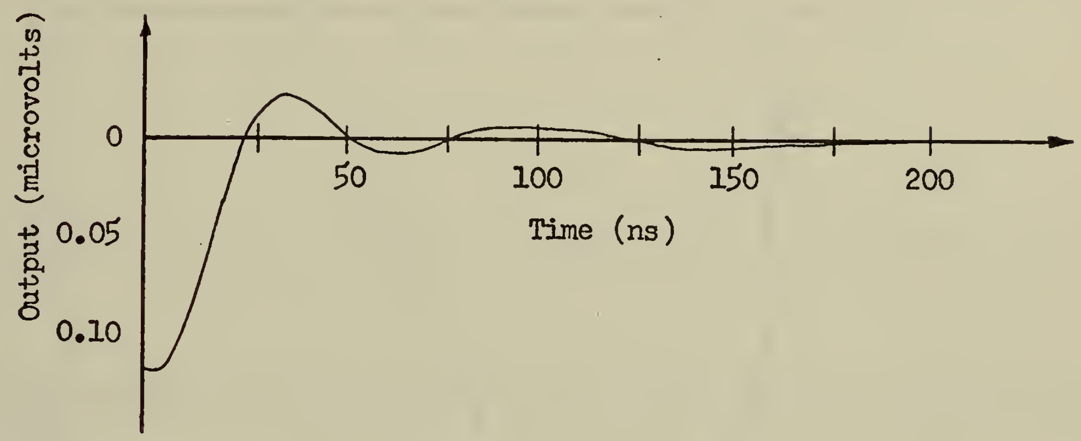


Figure 9. Impulse Response

Magnitude C_3 C_4 pf	Cutoff Frequency f_o MHz	Filter Passband Attenuation dbs	Time Delay maximum ns	Time Delay minimum ns
2.0	12.6	-80	2.73×10^2	2.7×10^{-6}
1.5	15.8	-80	2.64×10^2	3.4×10^{-7}
1.25	20.0	-80	2.56×10^2	4.0×10^{-7}
1.0	22.4	-80	negative	4.7×10^{-7}

Table 3. Results of Computer-Aided Analysis - Topological Approach

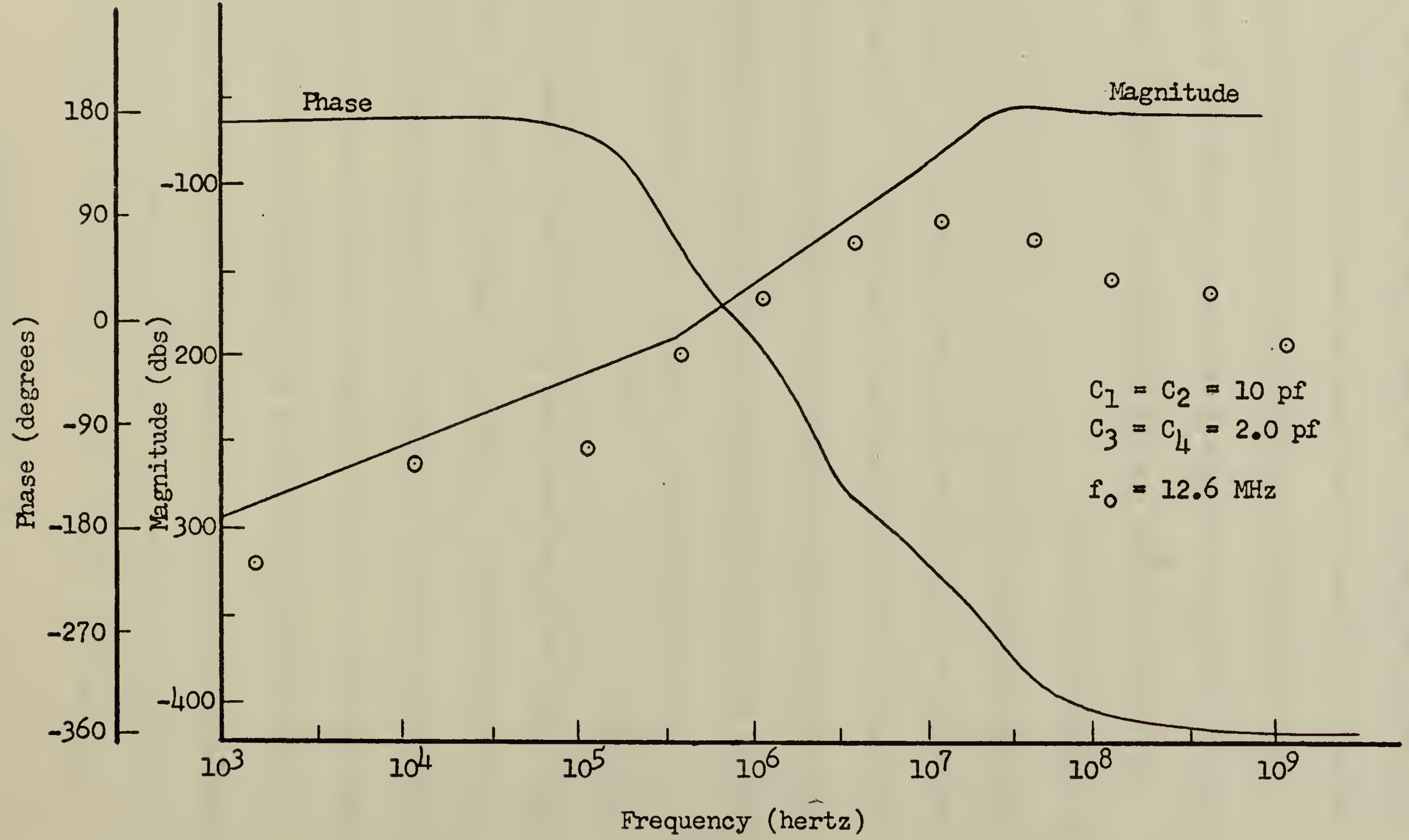


Figure 10. Frequency Response with Optimum Parameter and Actual Circuit (⊙)

IV. CIRCUIT PERFORMANCE

The original circuit, Fig. 1, less the output amplifier, was modified by changing the values of C_3 and C_4 to 2 picofarads. This value is the upper bound on the range of optimum values determined in the analysis.

The frequency response of the filter was then taken. The results were quite different than predicted and are plotted in Figure 10. The attenuation through the actual filter was on the average 10 to 15 db greater outside the passband. There was a definite downward slope in the passband. The cutoff frequency was also shifted to almost half of its calculated value.

A first the measurement techniques and instrumentation were suspect, but this proved to be false after careful review of procedures and equipment operation. Since the circuit was constructed on a "bread-board", stray and wiring capacitances were next thought to be the causes, and the circuit was transferred to printed circuit form to eliminate them. The frequency response was taken again. The results did not change enough to warrant another plot.

Next, other values for the capacitors, C_2 and C_4, were tried and the only significant changes noticed were outside the passband.

The minor adjustments suggested by Sallen and Key in Ref. 1 did not help to achieve the designed performance. The gains of the amplifiers were next altered to see their effect on the response characteristic. The gains were increased slightly by removing the collector resistors and decreased by increasing the collector resistance. There was some

improvement in the shape of the characteristics when the gain was increased.

The problem, it appears, goes a little deeper than the physical components and their minor adjustments.

V. CONCLUSIONS

The circuit would have performed as predicted if the active network constructed with physical components has possessed the assumed ideal parameters: infinite input impedance, zero output impedance, and highly stable gain. Referring to equations (9) and (10) on page 14, the damping factor d is directly related to the gains of the active devices. The damping factor influences the shape of the frequency response characteristics. The next step, if the analysis were to continue, would be to replace the junction transistors with field-effect types. The field-effect transistor is noted for its high input impedance. The output impedance is no greater than that of a junction transistor. The capacitances associated with the FET's are also small and no greater than those found in junction transistors.

An error in judgement was committed when the decision was mde to continue the analysis and synthesis at high frequencies without considering the effects of the junction capacitances. Had these factors been taken into account, the synthesis could have been more successful.

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13. ABSTRACT

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Two approaches were to be taken in the analysis and synthesis phase. Only one using topologically oriented programs was completed. The second involved the derivation and analysis utilizing the system transfer function.

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KEY WORDS	LINK A		LINK B		LINK C	
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Active filter						
Computer-aided-design						
Highpass						

Thesis
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The analysis and
synthesis of a fourth-
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